Assessment of Nonlinearity in Brain Electrical Activity: A DVV Approach

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Abstract

In this paper, we provide a comprehensive analysis of the EEG data recorded in a steady-state visual evoked potentials (SSVEPs) experiment. This is based upon the recently introduced the 'delay vector variance'(DVV) method for signal characterisation, which examines the local predictability of the signal in the phase space and checks simultaneously the signal nonlinearity and the determinism. Simulation results support the analysis.

1. Introduction

Brain-computer interfaces (BCIs) are technical systems which provide a direct connection between the human brain and a computer [1][2]. Research in this field has been increasing in the last few years. In particular, Middendorf *et al* [3] introduced a system that evaluates the focus of the subject's gaze by resulting amplitude changes of the measured steady-state visual evoked potentials (SSVEPs). A flickering light source elicits SSVEPs of the corresponding flickering frequency, measurable over the occipital cortex, while the subject shifts his gaze to these stimuli. In order to enable higher dimensional discrimination multiple flickering lights need to be introduced to . The analysis of these SSVEPs signals can help interpret the functioning of human brain. However, most of the methods are linear frequency domain based [4], which do not account for nonlinearity, multi-modal problems and high order statistics.

On the other hand, signal modality characterisation is becoming an increasingly important area of multi-disciplinary research and large effort has been put into devising efficient algorithms for this purpose. The idea is that the changes in the signal nature between linear and nonlinear and deterministic and stochastic can reveal information (knowledge) which is critical in certain applications, for instance, the nature of the heart rate variability signal changes from stochastic (linear) to chaotic (nonlinear) depending on whether the patient is healthy or not [5].

By the use of some recently introduced methods for signal characterisation, based on the local predictability in the state space, we therefore provide a theoretical analysis for EEG SSVEPs signals.

2. The Definition of 'Nature' of A Signal and 'Delay Vector Variance' (DVV) Method

By the signal 'nature', we adhere to the above two sets of signal properties: linear/nonlinear, and deterministic/stochastic. The strict definition of a *linear* signal is that it is generated by a linear shift-invariant system[1] driven by white Gaussian noise (WGN). In practice, this definition is relaxed somewhat by allowing the distribution of the signal to deviate from the Gaussian one, which can be interpreted as a linear signal following the strict definition, measured by a static (possibly nonlinear) observation function. A signal that cannot be generated in the above way is considered nonlinear. A signal is considered deterministic if it can be precisely described by a set of equations, otherwise it is considered stochastic.

There is one more concept we need to address before we introduce our novel method for signal characterisation, that is the method of surrogate data, or 'surrogates' for short. They are artificially generated randomised data, preserving the linear properties of the original signal, *e.g.*, mean, variance and power spectrum, while all the nonlinear predictability is destroyed during the randomisation. There are various methods for generating surrogate data. However, in this paper, we opt for iterative amplitude adjusted Fourier transform (iAAFT) surrogate data since it is proved to yield reliable results [6]. For more details on surrogate data, please refer to [7].

Many methods for detecting the nonlinear structure within a signal have been proposed, such as the classic 'surrogate

[1]System nonlinearity is defined differently from signal nonlinearity. A linear shift-invariant system, $f(\cdot)$, obeys the superposition and scaling property: i) for $a, b \in \mathbb{R} : f(ax+by) = af(x)+bf(y)$, ii) it produces identical outputs for a given input applied at different instants of time.

data' with different choices of discriminating statistics, 'deterministic versus stochastic' (DVS) plot [8], δ-ε Method [9]. For our purpose, it is desirable to have a method which is straightforward to visualise, and which makes use of some notions from nonlinear dynamics and chaos theory, *e.g.*, embedding dimension and phase space [10]. One such method is our recently proposed 'delay vector variance' (DVV) method [11], which is based upon examining the predictability of a signal in the phase space, and examines simultaneously the determinism and nonlinearity within a signal.

For an optimal[2] embedding dimension m, the DVV algorithm can be summarised in the following way:-

- Map the original time series from time domain to a set of delay vectors (DVs) in phase space, $\mathbf{x}(k) = [x_{k-\tau m}, \ldots, x_{k-\tau}]^T$, $k = 1, \ldots, N - m + 1$, where N denotes the length of the time series and τ denotes the time lag which for convenience is set to unity in all the simulations and the corresponding target x_k;

- The mean μ_d and standard deviation σ_d are computed over all pairwise Euclidean distances between DVs, $\|\mathbf{x}(i) - \mathbf{x}(j)\| (i \neq j)$;

- The sets $\Omega_k(r_d)$ are generated such that $\Omega_k(r_d) = \{\mathbf{x}(i) | \|\mathbf{x}(k) - \mathbf{x}(i)\| \leq r_d\}$, *i.e.*, sets which consist of all DVs that lie closer to $\mathbf{x}(k)$ than a certain distance

$$r_d(n) = \mu_d - n_d\sigma_d + (n-1)\frac{2n_d\sigma_d}{N_{tv} - 1}, \quad n = 1, \ldots, N_{tv} \tag{1}$$

  where N_{tv} denotes how fine the standardised distance is uniformly spaced, and n_d is a parameter controlling the span over which to perform the DVV analysis;

- For every set $\Omega_k(r_d)$, the variance of the corresponding targets, $\sigma_k^2(r_d)$, is computed. The average over all sets $\Omega_k(r_d)$, normalised by the variance of the time series, σ_x^2, yields the 'target variance', $\sigma^{*2}(r_d)$:

$$\sigma^{*2}(r_d) = \frac{\frac{1}{N}\sum_{k=1}^{N} \sigma_k^2(r_d)}{\sigma_x^2} \tag{2}$$

  We only consider a variance measurement *valid*, if the set $\Omega_k(r_d)$ contains at least $N_0 = 30$ DVs, since too few points for computing a sample variance yields unreliable estimates of the true variance. For more details, please refer to [11, 13].

The idea behind the DVV method is: if two DVs of a predictable signal lie close to one another in terms of their Euclidean distance, they should also have similar targets. The smaller the Euclidean distance between them, the more similar targets they have. Therefore, the presence of a strong deterministic component within a signal will lead to small target variances for small spans r_d. The minimal target variance, $\sigma^{*2}_{min} = min_{r_d}[\sigma^{*2}(r_d)]$, is a measure for the amount of noise present within the time series. Besides, the target variance σ^{*2}_{min} has an upper bound which is unity. This is because, when r_d becomes large enough, all DVs belong to the same set $\Omega_k(r_d)$. Thus, the variance of the corresponding target of those DVs will be almost identical to that of the original time series.

In the following step, the linear or nonlinear nature of the time series is examined by performing the DVV test on both the original and a number of surrogate time series[3] [14], using the optimal embedding dimension of the original time series.

Due to the standardisation of the distances, the DVV plots can be conveniently combined[4] within a *scatter diagram*, where the horizontal axis corresponds to the DVV plot of the original time series, and the vertical axis to that of the surrogate time series. If the surrogate time series yield DVV plots similar to that of the original time series as shown in Figure 1(a), the 'DVV scatter diagram' coincides with the bisector line, and the original time series is *judged to be linear*, as shown in Figure 1(c). If not as shown in Figure 1(b), the original time series is *judged to be nonlinear*, as depicted in Figure 1(d). Since the minimal target variance indicates a strong deterministic component within the signal, we conclude that in DVV scatter diagrams, the more the curve approaches the vertical axis, the more deterministic the nature of the signal. This can be employed as a convenient criterion for estimating the level of noise within a signal.

To illustrate the meaning of 'signal nature' and the usage of DVV method, consider a benchmark linear signal (AR(4)) [15], given by

$$\begin{aligned} x(k) &= 1.79x(k-1) - 1.85x(k-2) + 1.27x(k-3) \\ &\quad - 0.41x(k-4) + n(k) \end{aligned} \tag{3}$$

and a benchmark nonlinear signal, the Narendra Model Three

[2] In this paper, the optimal embedding dimension is calculated by Cao's method [12], since this method is demonstrated to yield robust results on various signals.

[3] In this paper, all the DVV tests are performed using 19 surrogate data realisations. The reason for this is that with the increase in the number of surrogate data, DVV tests do not yield a much better result whereas the computational complexity is much increased.

[4] In fact, target variance (σ^{*2}) of the original data is plotted against the mean of the target variance of 19 surrogate data, for all corresponding distances ($\frac{r_d - \mu_d}{\sigma_d}$).

[16], given by

$$z(k) = \frac{z(k-1)}{1+z^2(k-1)} + r^3(k)$$
$$r(k) = 1.79\, r(k-1) - 1.85\, r(k-2) + 1.27\, r(k-3) - 0.41\, r(k-4) + n(k) \qquad (4)$$

where $n(k) \sim \mathcal{N}(0,1)$. Figure 1 illustrates the signal nature of these two benchmark signals. The upper two diagrams are DVV plots, which are obtained by plotting the target variance as a function of standardised distance. The lower two diagrams (DVV scatter diagram) are obtained by plotting the target variance of the original data against the mean of the target variances of the surrogate data, where the error bars denote the standard deviation for the surrogate data. From the Figure, the DVV scatter diagram for AR(4) signal lies on the bisector line, indicating its linear nature whereas that for a Narendra Model Three signal deviates from the bisector line, indicating its nonlinear nature.

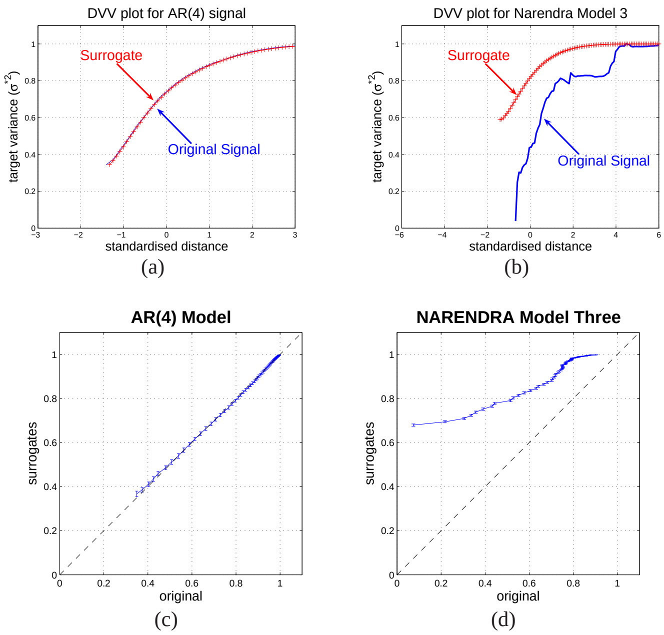

Figure 1: Nonlinear and deterministic nature of signals. The upper two diagrams represent DVV plots, obtained by plotting the target variance as a function of standardised distance, where the line with crosses denotes the DVV plot for the average of 25 iAAFT-based surrogates and the solid line denotes that for the original signal. The lower two diagrams represent DVV scatter diagrams, obtained by plotting the target variance of the original data against the mean of the target variances of the surrogate data where error bars denote the standard deviation of the target variance of surrogate data.

As it is capable of examining the nonlinearity and determinism simultaneously, DVV method is employed as a powerful tool for characterising the nature of a signal in the following chapters.

3. Experimental Settings and Simulation Results

The experiment of steady-state visual evoked potentials (SSVEPs) was conducted in the laboratory for advanced brain signal processing at RIKEN, Japan. Three different sets of EEG data were recorded during pre-stimuli, stimuli and post-stimuli sessions, when subjects were gazing at the center of computer screens where rectangles appeared from time to time.

The 'delay vector variance' (DVV) method was utilised to characterise these three EEG data sets in terms of nonlinearity and deterministic signal nature. The embedded dimension was found to be (do you have the parameter settings?) and time lag was set to unity in all the simulations. We chose to generate 25 iAAFT surrogate data sets since increasing the amount of surrogate data does not further improve the results but hugely increases the computation complexity.

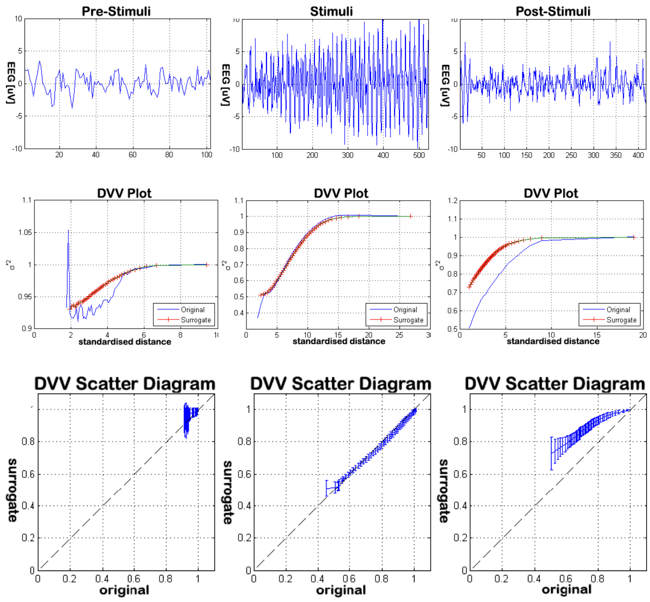

Figure 2: EEG data and associated DVV plots. Top panel: three EEG signals recorded pre-stimuli, during stimuli and post-stimuli from left to right. Middle panel: DVV plots for these three EEG signals, where the line with crosses denotes the DVV plot for the average of 25 iAAFT surrogates and the solid line denotes that for the original EEG data. Bottom panel: DVV scatter diagram for the three EEG signals, where the error bars denote the standard deviation for the average of the target variance Eq.(2) of surrogate data.

Figure 2 illustrates the DVV experiments conducted on the EEG data. Top panel denotes the time-domain representation of the three EEG signals recorded pre-stimuli, during stimuli and post-stimuli from left to right. The middle panel denotes DVV plots for these three EEG signals, where the line with crosses denotes the DVV plot for the average of 25 iAAFT surrogates and the solid line denotes that for the original EEG data. The bottom panel denotes DVV scatter diagram for the three EEG signals, where the error bars denote the standard deviation for the average of the target variance Eq.(2) of surrogate data.

From the top panel of the Figure, although there exists a difference between the stimuli EEG and the other two EEG data sets, we cannot tell whether it was caused due to the change of the nature of the EEG signal or simply the amplitude of the signal changed. However, in the DVV scatter diagrams for the three EEG signals (bottom panel), such difference was obvious and easy to observe. Bearing in mind that if the DVV scatter diagram lies on the bisector line, the signal is considered to be linear, the EEG signal during stimuli is almost linear whereas the other two are nonlinear. This can be understood in the following way:- During the stimuli session, the brain was doing nothing else but giving processing visual information the highest priority. Such focus results in the linear EEG signal. Before the stimuli appeared on the screen, the subject was probably wondering what would happen and the brain was not only processing the visual information but also information obtained from other perceptual organs, *e.g.*, ears, nose, which leads to the nonlinearity present in the EEG signal.

From the middle panel, the local predictability of those three EEG signals were examined by means of DVV plots. As discussed in Section 2, the value of the leftmost point of DVV plot indicates unpredictability, that is, the lower the more predictable. Clearly, the EEG signal recorded before stimuli has the worst local predictability (almost as unpredictable as white Gaussian noise), post-Stimuli EEG was slightly better and EEG for stimuli period was most predictable. This is because in pre-stimuli session the subject was inevitably anxious and nervous in some degree, the brain started to wonder and randomly process the information, which increased the noise level in the EEG signal. During the stimuli session, the brain started to function normally according to what the subject observed on the screen and the EEG signal appeared pseudo-periodical, which leads to the best predictability. However, in the post-stimuli session, although the subject stopped focusing, the images left during the stimuli session continued to appear in the brain, which increased the noise level in the EEG signal. Compared to the pre-stimuli session, the EEG signal is more predictable in post-stimuli session as the subject was not anxious or nervous any longer.

4. Conclusions

In this paper, we have provided a comprehensive analysis for the EEG signals recorded in a steady-state visual evoked potentials (SSVEPs) experiment. This is achieved by means of the recently introduced ‘delay vector variance’ (DVV) method for signal characterisation, based upon the local predictability in phase space. It has been shown that the EEG signal recorded during the stimuli session is linear and has better predictability than that for the other two sessions. Rigorous simulations support the analysis.

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