Signal Modality Characterisation Using Collaborative Adaptive Filters

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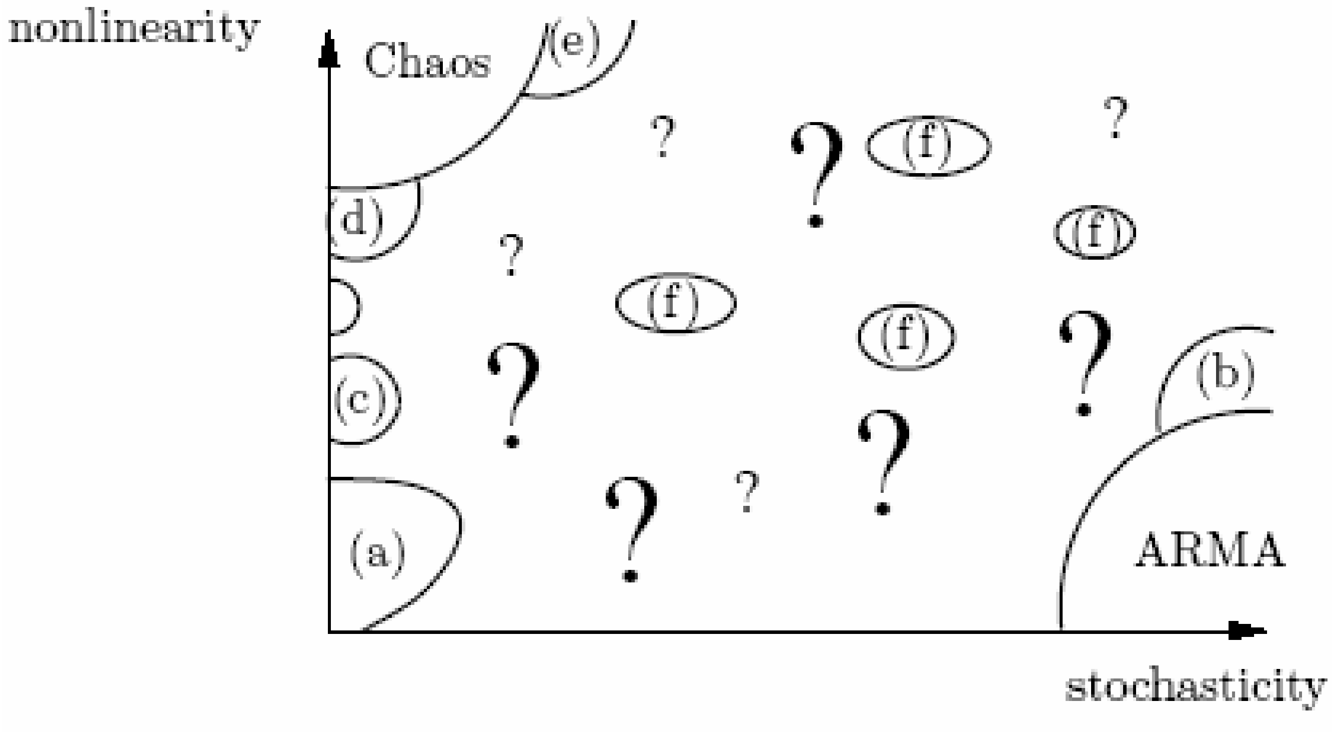
Signal Modality – General Perspective

Notice the difference between **Signal Nonlinearity and System Nonlinearity**

Deterministic vs. Stochatic nature or Linear vs. Nonlinear nature



Change in signal modality can indicate e.g. health hazard (fMRI, HRV)

Challenges in Signal Modality Characterisation

- Changes in the signal nature between (e.g. linear and nonlinear) can reveal **information** which is critical (e.g. health conditions);

- Existing algorithms based on **hypothesis testing** and operate in a batch manner;

- Other methods based on comparing outputs of **two adaptive filters of different kind** ⇒ choice of many parameters

- These filters **do not co–operate** ⇒ simple test but *non–unique solution*.

Our aim:- on–line signal modality characterisation for real–world problems

Benefits:- Synergy between the filters, existence and uniqueness of solution

Hybrid Filters

Key properties:-

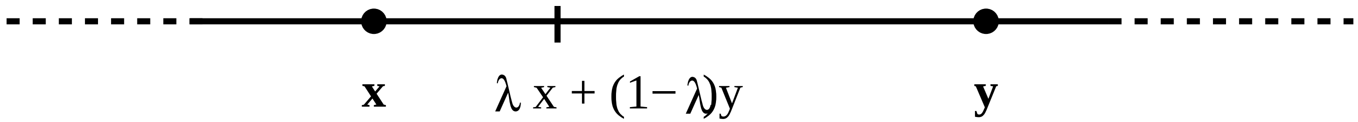
- Multiple individual adaptive subfilters operating in parallel;
- Subfilters feed into a mixing algorithm which produces the single output of the filter;
- Mixing algorithm is also adaptive and combines the outputs of the subfilters **(collaboration, synergy for two different filters**);

Advantages:-

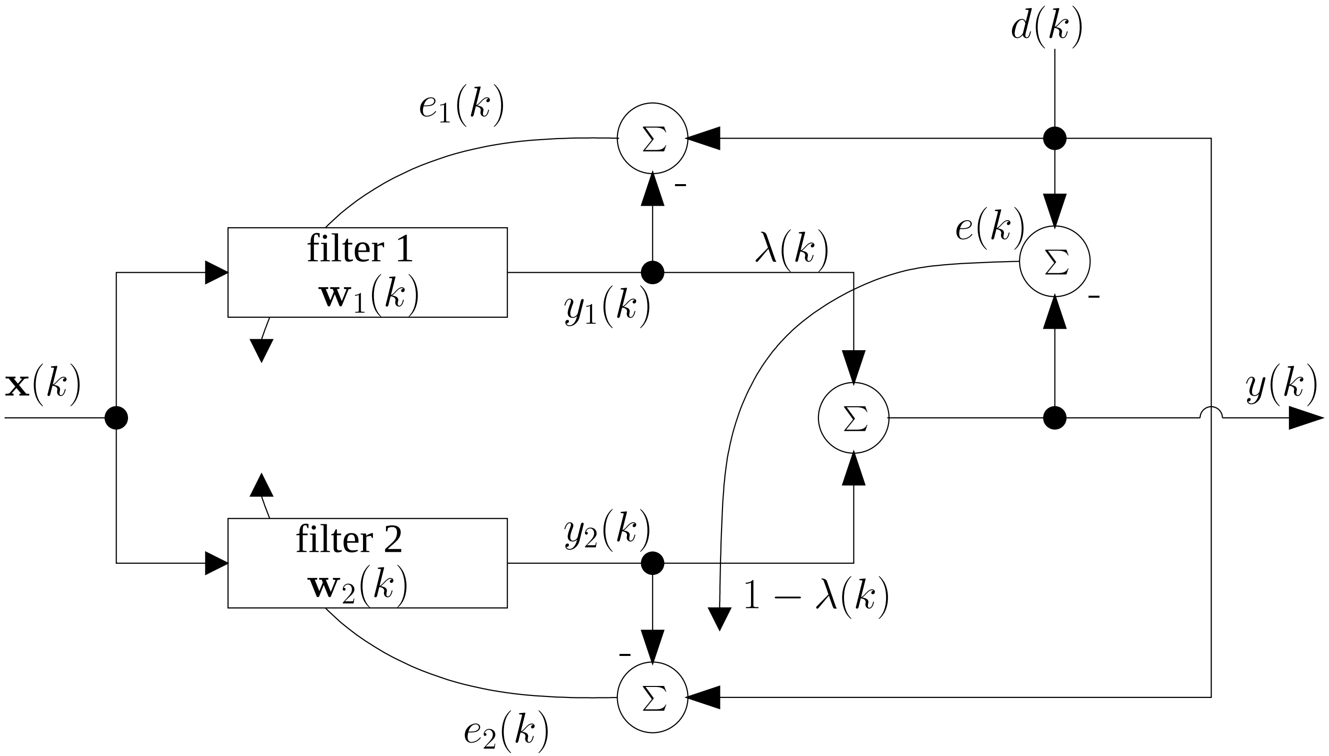
- When in "filtering mode", improved performance over the individual constituent filters;
- One effect of this mixing algorithm is that it can give an indication of which filter is currently responding to the input signal most effectively;
- By selecting algorithms which are suitable for either linear or nonlinear signals $\Rightarrow$ the mixing algorithm can adapt according to fundamental properties of the input signal.

Convex Hybrid Filtering Configuration

Virtues of Convex Combination ($\lambda \in [0,1]$)



Convexity $\Rightarrow$ existence and uniqueness of solution



Adaptation of Mixing Parameter λ (Modality Tracking)

To preserve the inherent characteristics of the subfilters, the constituent subfilters are each updated by their own errors $e_1(k)$ and $e_2(k)$, whereas the parameter λ is updated based on the overall error $e(k)$.

The convex mixing parameter $\lambda(k)$ is updated using the following gradient adaptation

$$\lambda(k+1) = \lambda(k) - \mu_\lambda \nabla_\lambda E(k)_{|\lambda=\lambda(k)}$$

where μ_λ is the adaptation step-size. The λ update can be shown to be

$$\begin{aligned}\lambda(k+1) &= \lambda(k) - \frac{\mu_\lambda}{2}\frac{\partial e^2(k)}{\partial \lambda(k)} \\ &= \lambda(k) + \mu_\lambda e(k)(y_1(k) - y_2(k))\end{aligned}$$

To ensure the combination of adaptive filters remains a convex function it is critical λ remains within the range $0 \leq \lambda(k) \leq 1$, a hard limit on the set of allowed values for $\lambda(k)$ was therefore implemented.

Nonlinear Hybrid Filter

The NLMS algorithm $\rightsquigarrow$ widely used, known for its robustness and excellent steady state properties

$$\begin{aligned}
y_{NLMS}(k) &= \mathbf{x}^T(k)\mathbf{w}_{NLMS}(k) \\
e_{NLMS}(k) &= d(k) - y_{NLMS}(k) \\
\mathbf{w}_{NLMS}(k+1) &= \mathbf{w}_{NLMS}(k) + \frac{\mu_{NLMS}}{\|\mathbf{x}(k)\|_2^2 + \varepsilon(k)} e_{NLMS}(k)\mathbf{x}(k)
\end{aligned}$$

The NNGD algorithm $\rightsquigarrow$ faster convergence speed and much better tracking capabilities

$$\begin{aligned}
y_{NNGD}(k) &= \Phi\left(net(k)\right) \\
net(k) &= \mathbf{x}^T(k)\mathbf{w}_{NNGD}(k) \\
e_{NNGD}(k) &= d(k) - y_{NNGD}(k) \\
\mathbf{w}_{NNGD}(k+1) &= \mathbf{w}_{NNGD}(k) + \eta(k)\Phi'\left(net(k)\right) e_{NNGD}(k)\mathbf{x}(k) \\
\eta(k) &= \frac{1}{C + \left[\Phi'\left(net(k)\right)\right] \|\mathbf{x}(k)\|_2^2}
\end{aligned}$$

Sparse Hybrid Filter

The NLMS algorithm $\rightsquigarrow$ proved a better choice than the LMS due to its improved tracking capabilities allowing it to adapt quickly to changes in the input signal preventing the sparse filter from dominating

The SSLMS algorithm $\rightsquigarrow$ specifically designed for sparse inputs

$$\begin{aligned} y_{SSLMS}(k) &= \mathbf{x}^T(k)\mathbf{w}_{SSLMS}(k) \\ e_{SSLMS}(k) &= d(k) - y_{SSLMS}(k) \\ \mathbf{w}_{SSLMS}(k+1) &= \mathbf{w}_{SSLMS}(k) + \mu\left(|\mathbf{w}_{SSLMS}(k)| + \varepsilon\right) e(k)\mathbf{x}(k) \end{aligned}$$

$\looparrowright$ **In both cases λ adapts according to the dynamics of the input**

Tracking Capability:- Synthetic Signals

The hybrid filters were evaluated in an adaptive one step ahead prediction setting with the length of the adaptive filters set to $N = 10$ for a set of 100 independent simulation runs.
The filters were presented with an input signal which alternated every 100 samples for the inputs described by a stable linear AR(4) process:

$$x(k) = 1.79x(k-1) - 1.85x(k-2) + 1.27x(k-3) - 0.41x(k-4) + n(k)$$

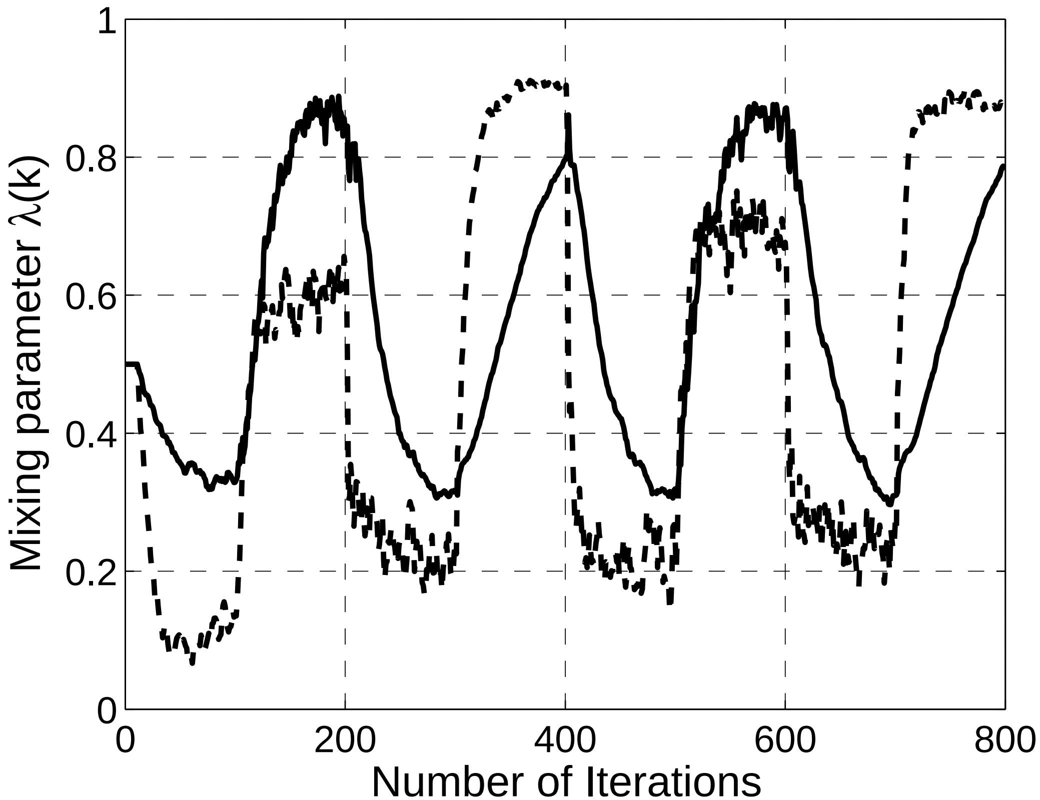
a benchmark nonlinear signal (Narendra III):

$$x(k+1) = \frac{x(k)}{1 + x^2(k)} + n^3(k)$$

and a benchmark sparse distribution, where $n(k)$ is a zero mean, unit variance white Gaussian process.
$\lambda = 1$ corresponds to the output of the NNGD/SSLMS trained subfilters and $\lambda = 0$ corresponds to the output of the NLMS trained subfilters.

Tracking Capability:- Synthetic Signals

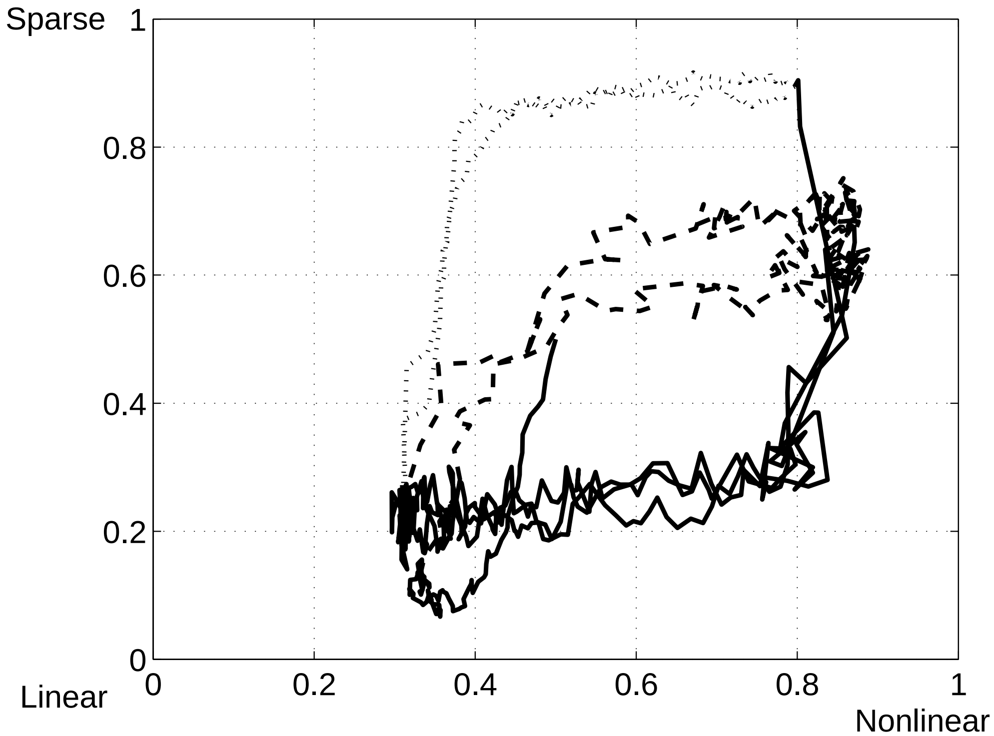
The evolution of the mixing parameters, nonlinear hybrid filter (solid line) and sparse hybrid filter (broken line)



⇒ output of the convex combination is dominated by the filter most appropriate for the input signal characteristics

Tracking Multiple Modality Changes

Comparison of the evolution of the mixing parameters, linear sections (solid line), nonlinear sections (broken line) and sparse sections (dotted line)



⇒ allows tracking of multiple different signal characteristics

Tracking Multiple Modality Changes

- correlation between the evolution of the two λs during the linear sections, differences in responses can be attributed to differences in learning rates;

- for the nonlinear and the sparse signals, however, the sparsity and saturation type nonlinearity are different phenomena and the sparse and nonlinear filter behaved differently;

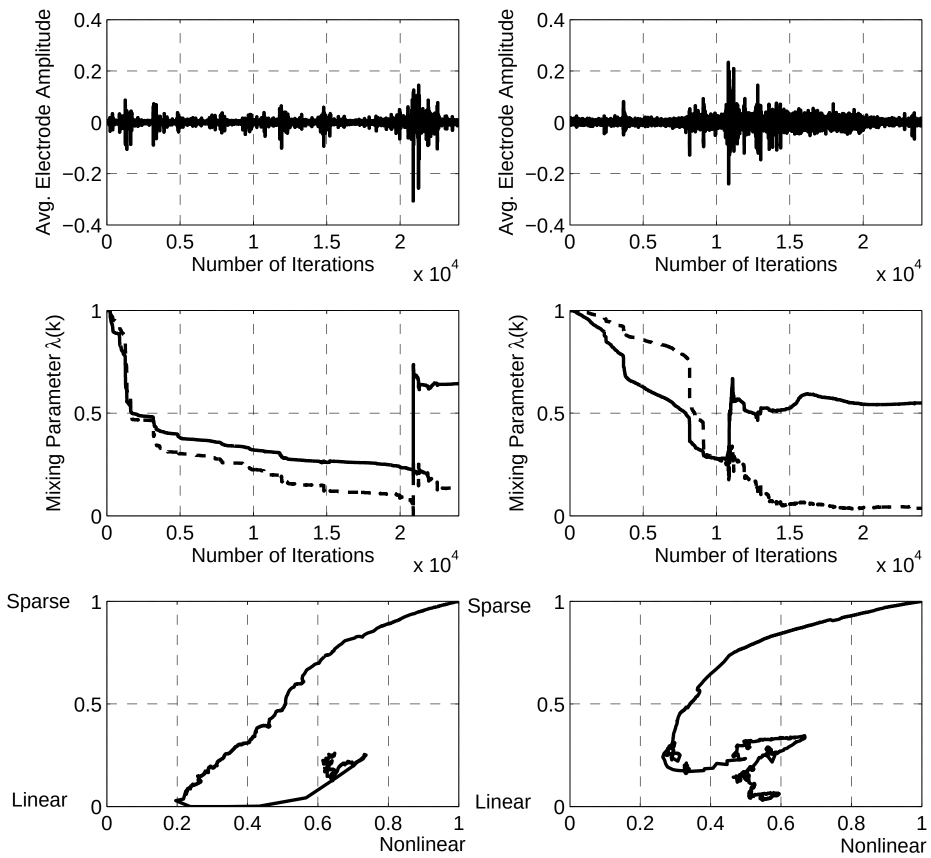
- as expected (as sparsity can be considered a subset of nonlinearity) the nonlinear hybrid filter obtains similar results for both the nonlinear and sparse inputs;

- the sparse hybrid filter shows a marked difference in levels of sparsity for the same inputs.

$\Rightarrow$ These results highlight the use of this technique in building a complete understanding of the nature of signals

Real-World Applications:- Epileptic Seizure Data

EEG data showing the onset of epileptic seizures has been observed



The proposed approach effectively detects changes in the nature of the EEG signals

Conclusions

- Novel approach to identify changes in the modality of a signal;
- Convex combination of two adaptive filters for which the transient responses are significantly different, in order to exploit the different performance capabilities of each;
- Collaborative adaptive signal processing approach, based on synergy between the constitutive filters;
- The evolution of the adaptive convex mixing parameter λ, helps determine which filter is more suited to the current input signal dynamics, and thereby gain information about the nature of the signal;
- The analysis and simulations illustrate that there is significant potential for the use of this method for online tracking of some fundamental properties of the input signal.

References

1. J. Arenas-Garcia, A.R. Figueiras-Vidal & A.H. Sayed, "Mean-square performance of a convex combination of two adaptive filters," *IEEE Trans. Signal Proc.*, vol. 54, no. 3, pp. 1078–1090, 2006.
2. B. Jelfs, P. Vayanos, S. Javidi, V.S.L. Goh & D.P. Mandic, "Collaborative Adaptive Filters for Online Knowledge Extraction and Information Fusion", *Signal Processing Techniques for Knowledge Extraction and Information Fusion*. Springer; 2007.
3. S.S. Kozat & A.C. Singer, "Multi-stage adaptive signal processing algorithms," *Proc. IEEE Sensor Array & Multichannel Signal Proc. Workshop*, pp. 380–384, 2000.
4. D. Mandic, P. Vayanos, C. Boukis, B. Jelfs, S.L. Goh, T. Gautama & T. Rutkowski, "Collaborative Adaptive Learning using Hybrid Filters," *Proc. IEEE Int. Conf. Acoustics, Speech & Signal Proc., ICASSP*, vol. 3, pp. 921–924, 2007.
5. D.P. Mandic, "NNGD algorithm for neural adaptive filters," *Electronics Lett.*, vol. 36, no. 9, pp. 845–846, 2000.
6. R. Martin, W. Sethares, R. Williamson, & C. Johnson, Jr., "Exploiting sparsity in adaptive filters," *IEEE Trans. Signal Proc.*, vol. 50, no. 8, pp. 1883–1894, 2002.
7. K.S. Narendra & K. Parthasarathy, "Identification and control of dynamical systems using neural networks," *IEEE Trans. Neural Networks*, vol. 1, no. 1, pp. 4–27, 1990.
8. B. Widrow & S.D. Stearns, *Adaptive Signal Processing*, Prentice-Hall; 1985.