Collaborative Adaptive Filtering in the Complex Domain

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Introduction

- Processing in the complex domain provides a natural processing platform (phase, synchrony, analytic signals).
- Complex–valued signals can be either complex by design (communications) or by convenience of representation (radar, sonar).
- Processing real domain signals in $\mathbb{C}$ allows the inclusion of phase components, resulting in multidimensional solutions with benefits over real domain solutions [1].

Complex Least Mean Square

The CLMS [2]:

$$
\begin{aligned}
e(k) &= d(k) - \mathbf{x}^T(k)\mathbf{w}(k), \\
\mathbf{w}(k+1) &= \mathbf{w}(k) + \mu e(k)\mathbf{x}^*(k),
\end{aligned}
$$

- is a natural extension of the LMS algorithm;
- benefits from stabitliy & robustness of LMS;
- allows simultaneous filtering of the real and imaginary parts of complex-valued data.

$\looparrowright$ However the CLMS does not take account of augmented complex statistics

Augmented Complex Statistics

Augmented complex statistics takes into account not only the “standard” covariance matrix but also the pseudo–covariance matrix [3]

$$\mathcal{C}_{\mathbf{xx}} = E\{\mathbf{x}\mathbf{x}^H\}, \;\; \mathcal{P}_{\mathbf{xx}} = E\{\mathbf{x}\mathbf{x}^T\}$$

For circular complex processes $\mathcal{P}_{\mathbf{xx}} = 0$.
For the generality of complex signals (non–circular) taking into account only the covariance matrix and not the pseudo–covariance matrix results in undermodelling.

Augmented CLMS

The ACLMS [4] utilises the full second order statistical information available within the signal by using an augmented complex vector as its input.

$$e(k) = d(k) - \mathbf{x}_a^T(k)\mathbf{w}(k)$$
$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu e(k)\mathbf{x}_a^*(k).$$

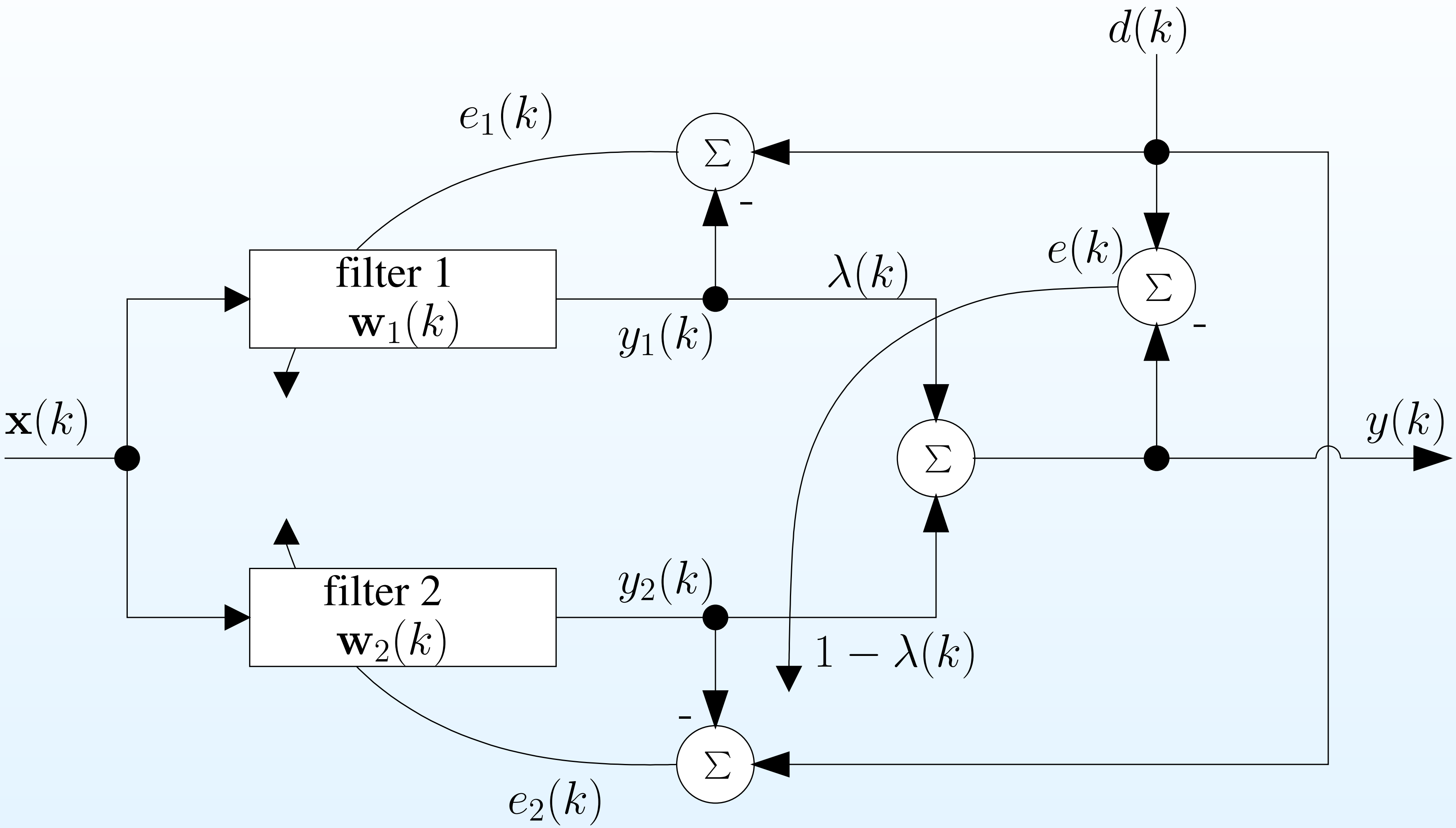
where $\mathbf{x}_a$ and its covariance matrix $\mathcal{C}_{\mathbf{x_a x_a}}$ are

$$\mathbf{x}_a = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}^* \end{bmatrix} \quad \mathcal{C}_{\mathbf{x_a x_a}} = \begin{bmatrix} \mathcal{C}_{\mathbf{xx}} & \mathcal{P}_{\mathbf{xx}} \\ \mathcal{P}^*_{\mathbf{xx}} & \mathcal{C}^*_{\mathbf{xx}} \end{bmatrix}$$

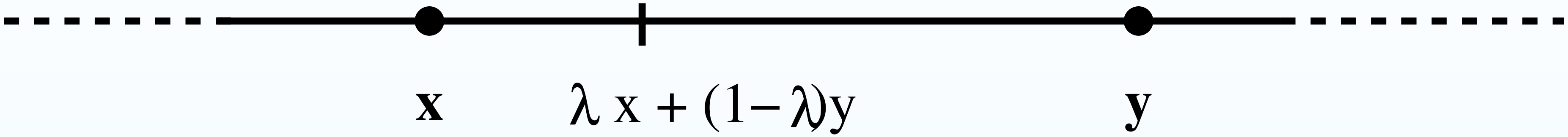
Hybrid Filters

- Hybrid filters have been introduced to improve the performance and stability of adaptive filters [5].
- We propose a hybrid filter which, following the approach from [6], combines the output of the CLMS with that of the ACLMS in a convex manner.
- By combining the ACLMS and CLMS our aim is to design a filter with better overall characteristics for both circular and non–circular complex signals than either of the individual algorithms.

Hybrid Filter Structure



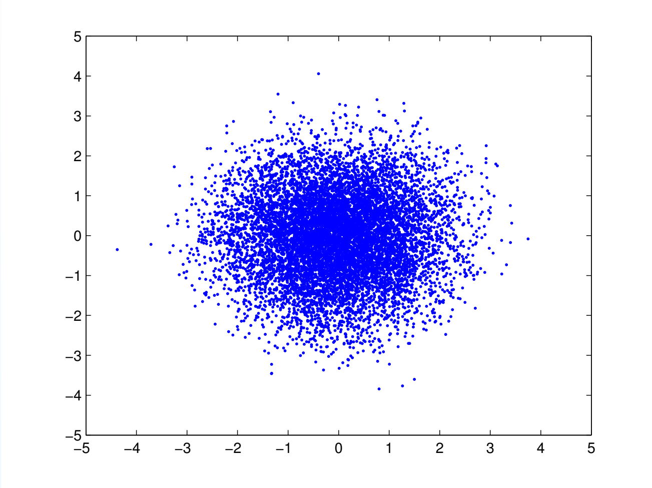
Mixing Parameter Update



Convexity $\Rightarrow$ existence of solution

$$
\begin{aligned}
y(k) &= \lambda(k) y_1(k) + (1-\lambda(k)) y_2(k) \\
\lambda(k+1) &= \lambda(k) - \mu_\lambda \nabla_\lambda E(k)|_{\lambda=\lambda(k)} \\
\nabla_\lambda E(k)|_{\lambda=\lambda(k)} &= \left\{ e(k)\frac{\partial e^*(k)}{\partial \lambda(k)} + e^*(k)\frac{\partial e(k)}{\partial \lambda(k)} \right\} \\
&= \Re\left\{ e(k)\left(y_1(k) - y_2(k)\right)^* \right\} \\
\lambda(k+1) &= \lambda(k) + \mu_\lambda \Re\left\{ e(k)\left(y_1(k) - y_2(k)\right)^* \right\}
\end{aligned}
$$

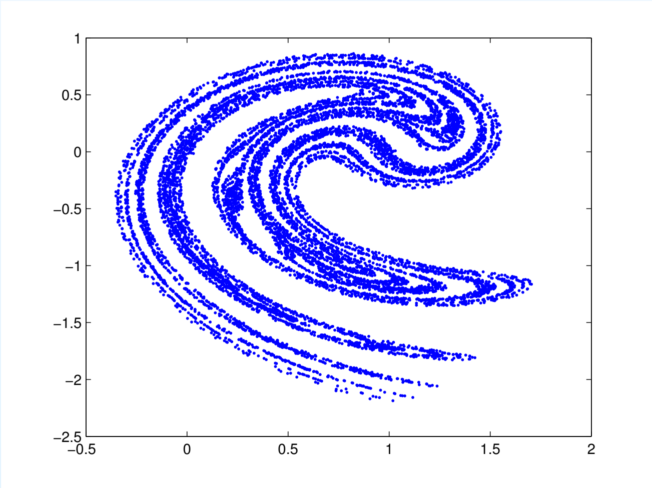
Circular & Non–Circular Data



The circular signal used was a stable AR(4) process

$$x(k)=1.79x(k-1)-1.85x(k-2)+1.27x(k-3)$$
$$-0.41x(k-4)+n(k)$$

$$n(k)=\rho(k)\cos(\theta(k))+j\rho(k)\sin(\theta(k))$$



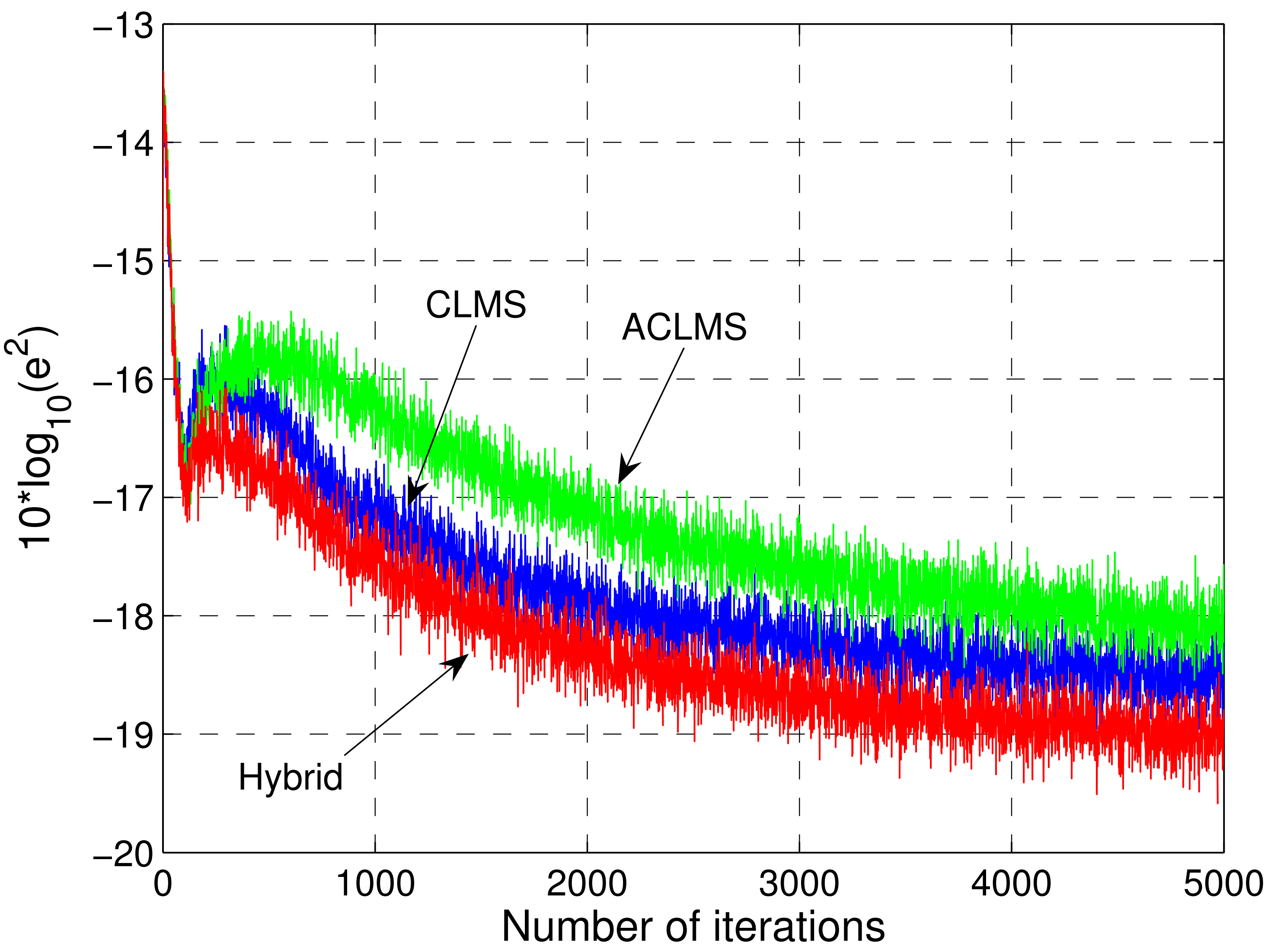
The non–circular data used was the Ikeda map [7]

$$x(k+1)=1+u[x(k)\cos t(k)-y(k)\sin t(k)]$$

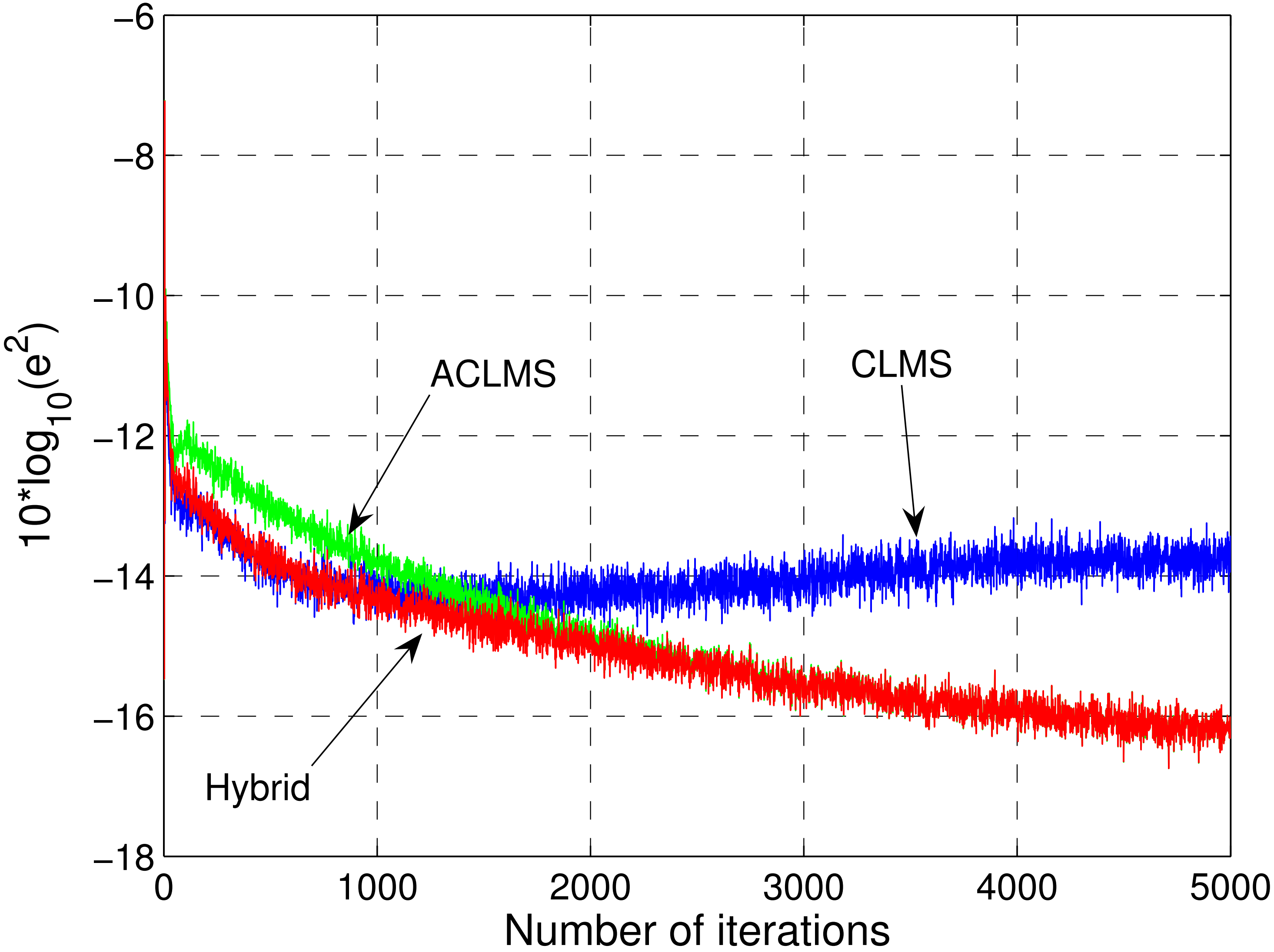
$$y(k+1)=u[x(k)\sin t(k)+y(k)\cos t(k)]$$

$$t(k)=0.4-\frac{6}{1+x^2(k)+y^2(k)}$$

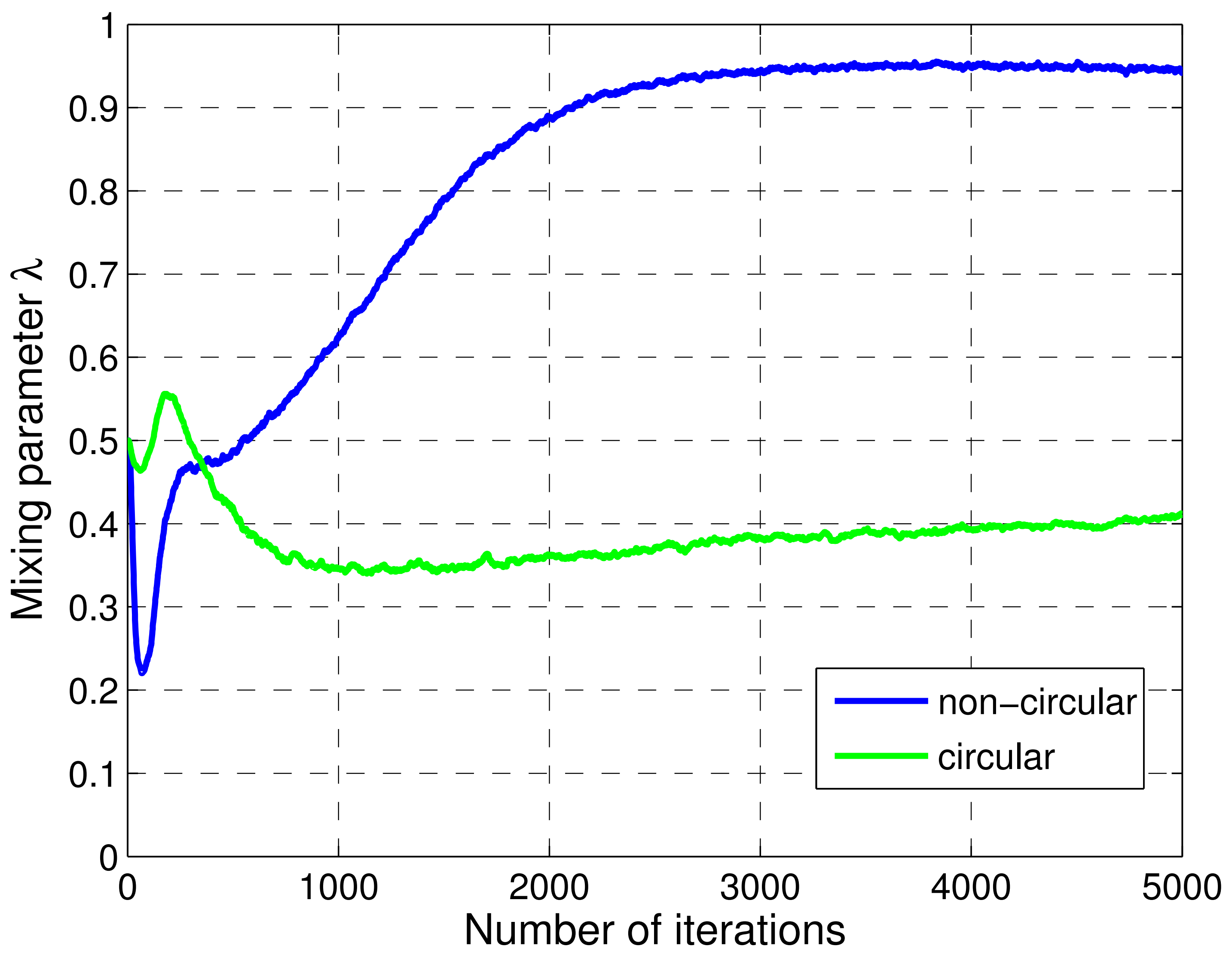
Convergence Curves - Circular



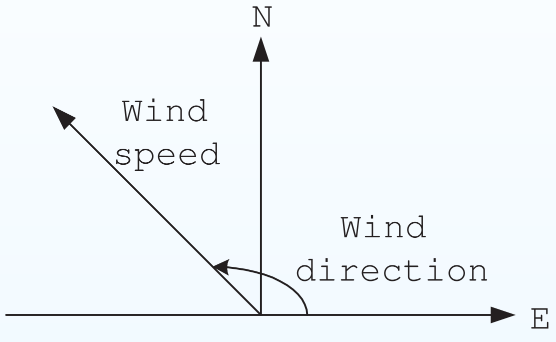
Convergence Curves - Non-Circular



Evolution of the mixing parameter



Prediction of Wind



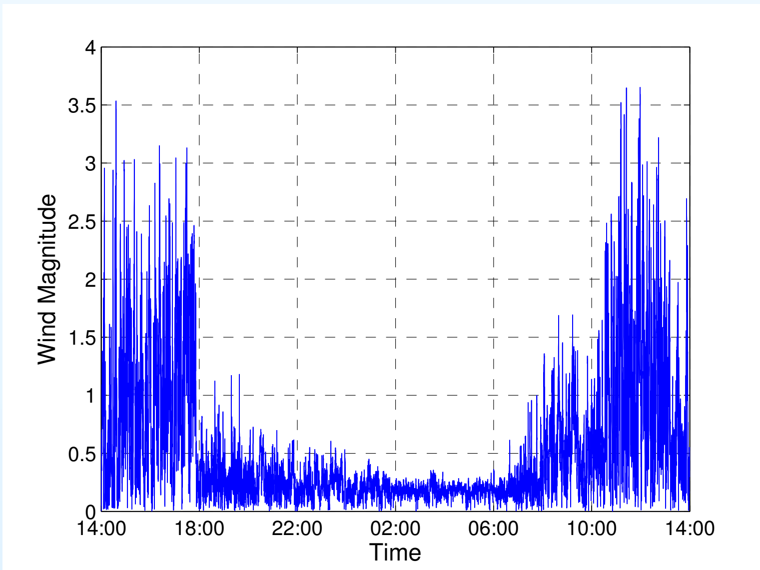
It is clear that the wind vector $\mathbf{v}$ can be represented in the complex domain as $\mathbf{V} = \mathbf{v} \cdot e^{j\theta}$ where $\mathbf{v}$ is the speed and θ the direction.

The wind data was measured over a 24 hour period.

The wind speed readings were taken in the north–south ($\mathbf{V}_N$) and east–west ($\mathbf{V}_E$) directions where

$$\mathbf{v} = \sqrt{\mathbf{V}_E^2 + \mathbf{V}_N^2}$$

$$\theta = \arctan\left(\frac{\mathbf{V}_N}{\mathbf{V}_E}\right)$$



Prediction Gains

Prediction gain $R_p = 10 \log_{10} \frac{\sigma_y^2}{\sigma_e^2}$ for the CLMS, ACLMS and hybrid filters for wind data and circular and non–circular synthetic data

	AR(4)	Ikeda	'Calm' wind	'High' wind
CLMS	5.25	0.65	7.03	3.26
ACLMS	4.73	**3.77**	6.87	4.35
Hybrid	**5.66**	3.73	**7.33**	**4.48**

Conclusions

- A hybrid filter consisting of a convex combination of the CLMS and ACLMS algorithms has been introduced.
- The hybrid filter takes advantage of the faster convergence speeds of the CLMS and the improved performance in the steady state for non–circular data of the ACLMS.
- It has been shown that the hybrid filter can outperform both of the subfilters for synthetic circular and non–circular data.
- A real world wind signal has been used to demonstrate that for signals where the nature may be changing the hybrid filter will perform consistently well.

References

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