Fuzzy Entropy based Nonnegative Matrix Factorization for Muscle Synergy Extraction

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Motivation

We propose a new nonnegative matrix factorization algorithm which employs a cross fuzzy entropy similarity measure to extracting muscle synergies which preserve the complexity of the recorded muscular data

- Muscle synergies are common patterns of muscle activations which serve as building blocks to produce detailed movements, reducing the number of degrees of freedom to be controlled [1]
- Requires an accurate method for extracting the synergies which can reconstruct data capable of producing movement which could complete the required task
- Entropy is commonly used in classification and assessment of changes in muscle activity recorded via surface electromyography (EMG) [2]

Nonnegative Matrix Factorization

The standard approach to muscle synergy extraction is to use a nonnegative matrix factorization (NMF)

That is, given

- data matrix $Y \in \mathbb{R}^{X\times N}$
- positive integer $K < \min\{X, N\}$

find nonnegative matrices

- $W \in \mathbb{R}^{X\times K}$
- $H \in \mathbb{R}^{K\times N}$

achieved via

$$\min_{W,H} D(Y\|WH) \quad \text{subject to} \quad W \geq 0, \quad H \geq 0$$

where $D(Y\|WH)$ is a measure of goodness of fit

$$\underset{(X \times N)}{Y} \approx \underset{(X \times K)}{W}\ \underset{(K \times N)}{H}$$

EMG Data

The EMG data has previously been described in [4], in brief:

- 5 healthy right-handed subjects
- Each subject grasped a cylindrical object the movement was repeated 3 times

Muscles recorded and location over muscle for muscles with multiple recordings
Upper indicating closer to the body (origin) and lower indicating closer to the hand (insertion).

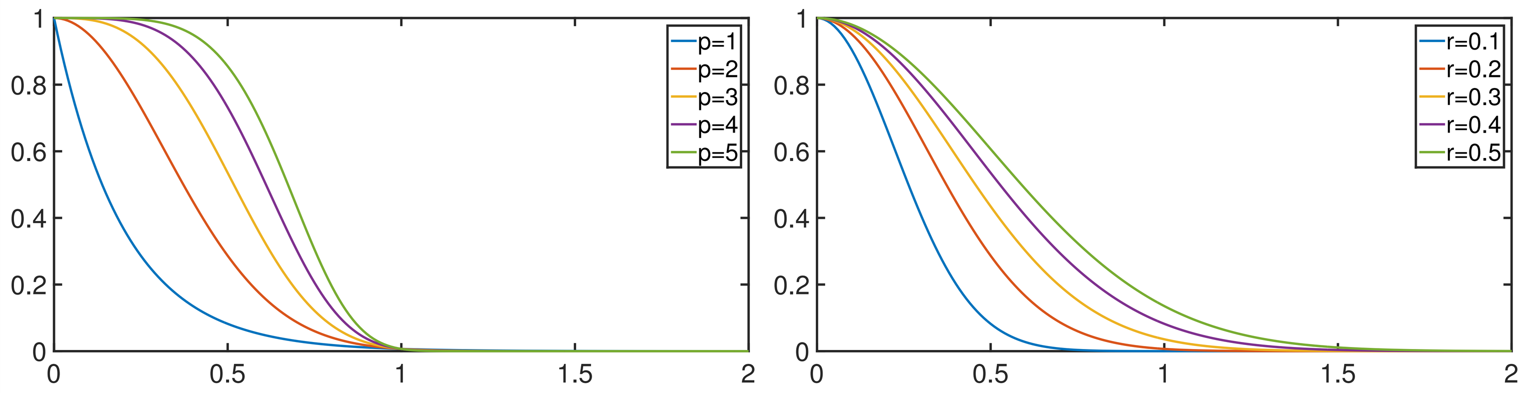
Ch.	Muscle	Location	Ch.	Muscle	Location
1	Extensor digitorum	upper	9	Extensor carpi ulnaris	
2	Anconeus		10	Extensor digitorum	lower
3	Flexor carpi ulnaris		11	Extensor carpi radialis brevis	
4	Pronator teres	lower	12	Extensor carpi radialis longus	
5	Flexor carpi radialis	upper	13	Abductor pollicis brevis	
6	Flexor carpi radialis	lower	14	Abductor digiti minimi	
7	Palmaris longus		15	Biceps brachii	upper
8	Pronator teres	upper	16	Biceps brachii	lower

Fuzzy Entropy

C-FuzzyEn employs an exponential function to give a continuous degree of similarity between vectors based on their closeness [3].

The degree of similarity between any pair of vectors is defined in terms of a fuzzy function of the distance d between them.

$$D^m_{i,j}(p, r) = \exp\left(-\left(d^m_{i,j}\right)^p / r\right)$$



Effects of p (for $r = 0.2$) & r (for $p = 2$) on the fuzzy boundary

The overall similarity function for vectors of length m, ϕ^m, then becomes

$$\phi^m(p, r) = \frac{1}{N-m}\sum_{i=1}^{N-m}\left(\frac{1}{N-m}\sum_{j=1}^{N-m} D^m_{ij}\right)$$

The C-FuzzyEn of the two time series can be expressed in terms of the ratio of the negative log of the conditional probability ϕ^{m+1}/ϕ^m

For a finite time series this is equivalent to

$$\boxed{\textbf{C-FuzzyEn}(\boldsymbol{m, p, r, N}) = \ln\boldsymbol{\phi^m(p, r)} - \ln\boldsymbol{\phi^{m+1}(p, r)}}$$

Proposed NMF Update

We define the measure of goodness of fit $D(Y\|WH)$ in the NMF algorithm in terms of the C-FuzzyEn of the elements of Y and $W \cdot H$.

Then using a gradient descent rule, we have the updates

$$[W]_{x,k} \leftarrow W_{x,k} - \eta^W \nabla_{[W]_{x,k}}\Big(D\big(Y\|WH\big)\Big)$$
$$[H]_{k,n} \leftarrow H_{k,n} - \eta^H \nabla_{[H]_{k,n}}\Big(D\big(Y\|WH\big)\Big)$$

where η^W and η^H are the respective learning rates.

For simplicity, the gradient of the cost function is taken in terms of the constituent parts of the cost function, $\ln\phi^m$ and $\ln\phi^{m+1}$

For the update of W the gradient is taken row-wise with respect to the individual elements of W

$$\nabla_{[W]_{x,k}} = \nabla_{W_{x,k}} \ln\phi^m_x - \nabla_{W_{x,k}} \ln\phi^{m+1}_x$$

In contrast the gradient of the cost function is taken column-wise with respect to the elements of H,

$$\nabla_{[H]_{k,n}} = \nabla_{H_{k,n}} \ln\phi^m_n - \nabla_{W_{x,k}} \ln\phi^{m+1}_n$$

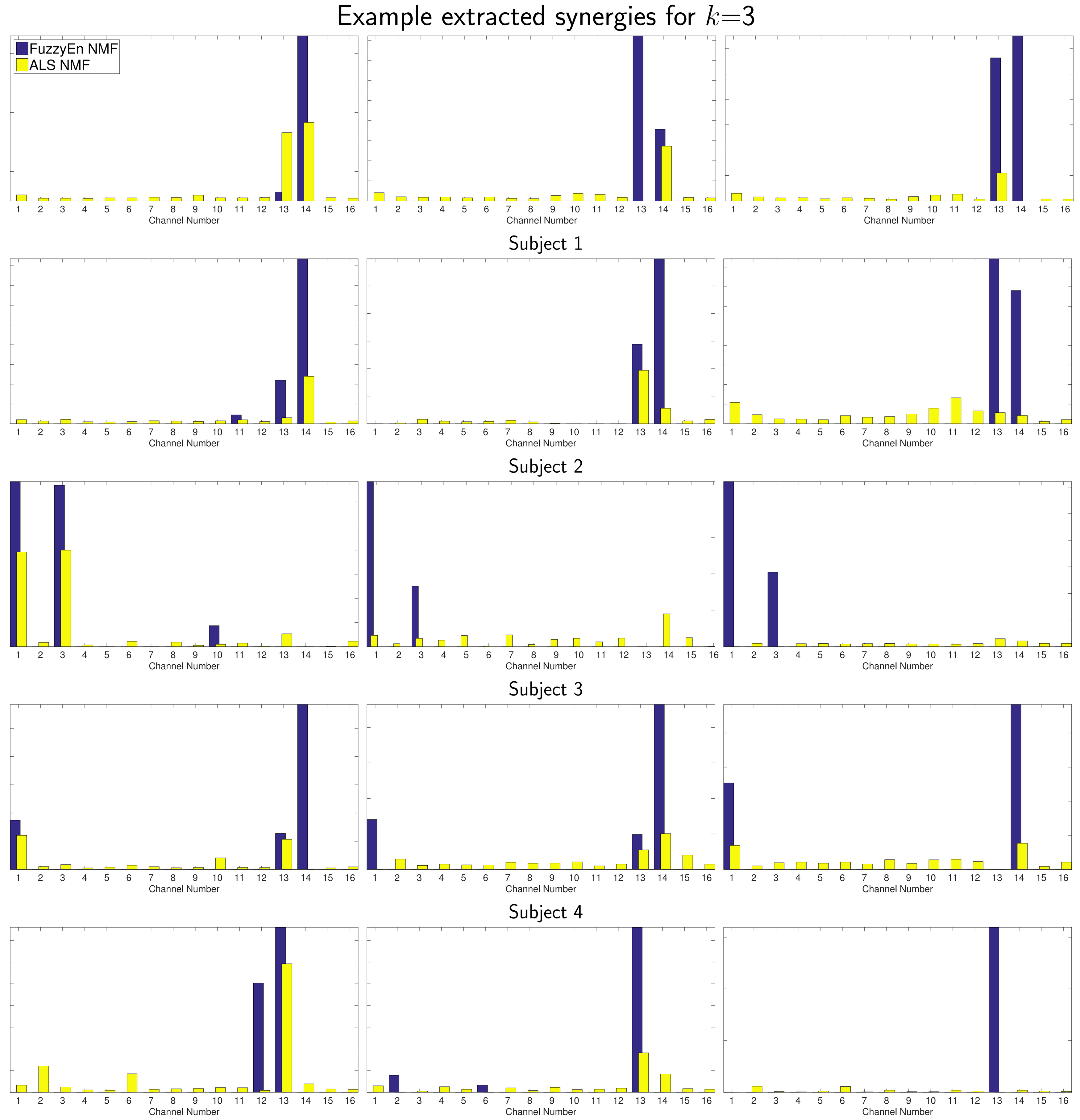
Substituting into the updates of W and H respectively gives us the NMF update with a C-FuzzyEn based cost function distance measure.

Algorithm Performance

Comparison of the average AIC for the proposed FuzzyEn NMF and the ALS NMF across a range of number of synergies.

Algorithm	Number of Synergies												
$\times 10^3$	3	4	5	6	7	8	9	10	11	12	13	14	15
ALS	9.397	12.528	15.659	18.790	21.992	25.053	28.184	31.316	34.448	37.579	40.711	43.848	46.977
FuzzyEn	9.390	12.521	15.652	18.783	21.915	25.045	28.177	31.308	34.439	37.570	40.702	43.831	46.963

Example extracted synergies for k=3



References

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